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DIRICHLET AVERAGE OF NEW GENERALIZATION OF GENERALIZED M-SERIES AND FRACTIONAL DERIVATIVE

Manoj Sharma
* Department of Mathematics RJIT, BSF Academy, Tekanpur

ABSTRACT

In this paper Dirichlet average of a new Special function called as New generalization of Generalized M-series [17] which is recently given by Ahmad Faraj , Tariq Salim , Safaa Sadek, Jamal Ismai [10] has been obtained. This series is a particular case of Fox's H-function and it is a Generalized case of Generalized M-series [6] defined by Author. The New Generalized M-series is interesting because the ${}_pF_q$ -hyper geometric function and the Generalized M-Series follow as its particular cases and these functions have recently found essential applications in solving problems in physics, biology, engineering and applied sciences.

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INTRODUCTION

Carlson [1-5] has defined Dirichlet average of functions which represents certain type of integral average with respect to Dirichlet measure. He showed that various important special functions can be derived as Dirichlet averages for the ordinary simple functions likex^t,e^x etc. He has also pointed out [3] that the hidden symmetry of all special functions which provided their various transformations can be obtained by averaging x^n,e^x etc. Thus he established a unique process towards the unification of special functions by averaging a limited number of ordinary functions. Almost all known special functions and their well known properties have been derived by this process.

In this paper the Dirichlet average of a new Special function called as New Generalized M-series has been obtained.

DEFINITIONS

We give blew some of the definitions which are necessary in the preparation of this paper.

Standard Simplex in $R^n, n \geq 1$

We denote the standard simplex in R^n, $n \geq 1$ by [1, p.62].

$$E = E_n = \{S(u_1, u_2, \dots \dots . . u_n) \;:\; u_1 \geq 0, \dots \dots \dots . u_n \geq 0,\; u_1 + u_2 + \cdots \dots \dots + u_n \leq 1\} \qquad (2.1.1)$$

Dirichlet measure

Let $b \in C^k, k \geq 2$ and let $E = E_{k-1}$ be the standard simplex in R^{k-1}. The complex measure μ_b is defined by $E[1]$.

$$d\mu_b(u) = \frac{1}{B(b)} u_1^{b_1-1} \dots \dots \dots \dots . u_{k-1}^{b_{k-1}-1} (1 - u_1 - \cdots \dots \dots - u_{k-1}) b_k^{-1} du_1 \dots \dots \dots . du_{k-1} \qquad (2.2.1)$$

Will be called a Dirichlet measure.
Here

$$B(b) = B(b1, \dots \dots . bk) = \frac{\Gamma(b_1) \dots \dots \dots \dots . . \Gamma(b_k)}{\Gamma(b_1 + \cdots \dots \dots . . + b_k)},$$

$$C_> = \{z \in z : z \neq 0, |ph\, z| < {}^{\pi}/_{2}\},$$

Open right half plane and $C_>$k is the k^{th} Cartesian power of $C_>$

Dirichlet Average[1, p.75]

Let Ω be the convex set in $C_>$, let $z = (z_1, \dots \dots \dots , z_k) \in \Omega^k$, $k \geq 2$ and let $u.z$ be a convex combination of $z_1, \dots \dots \dots , z_k$. Let f be a measureable function on Ω and let μ_b be a Dirichlet measure on the standard simplex E in R^{k-1}.Define

$$F(b,z) = \int_E f(u.z)d\,\mu_b(u) \tag{2.3.1}$$

We shall call F the Dirichlet measure of f with variables $z = (z_1, \dots \dots \dots, z_k)$ and parameters $b = (b_1, \dots \dots \dots . b_k)$.
Here

$$u.z = \sum_{i=1}^{k} u_i z_i \text{ and } u_k = 1 - u_1 - \cdots \dots \dots . - u_{k-1} \tag{2.3.2}$$

If $k = 1$, define $F(b,z) = f(z)$.

Fractional Derivative [8, p.181]

The concept of fractional derivative with respect to an arbitrary function has been used by Erdelyi[8]. The most common definition for the fractional derivative of order α found in the literature on the "Riemann-Liouville integral" is

$$D_z^{\alpha} F(z) = \frac{1}{\Gamma(-\alpha)} \int_0^z F(t)(z-t)^{-\alpha-1} dt \tag{2.4.1}$$

Where $Re(\alpha) < 0$ and $F(x)$ is the form of $x^p f(x)$, where $f(x)$ is analytic at $x = 0$.

THE NEW GENERALIZED M-SERIES

Here , first the notation and the definition of the New Generalized M-series, introduced by Ahmad Faraj , Tariq Salim , Safaa Sadek, Jamal Ismai [10] has been given as

$$\underset{p,q;m,n}{\overset{\alpha,\beta}{M}}(a_1 \dots, a_p; b_1, \dots, b_q; z) = \underset{p,q;m,n}{\overset{\alpha,\beta}{M}}(z),$$

$$\underset{p,q;m,n}{\overset{\alpha,\beta}{M}}(z) = \sum_{k=0}^{\infty} \frac{(a_1)_{km} \dots (a_p)_{km}}{(b_1)_{kn} \dots (b_q)_{kn}} \frac{z^k}{\Gamma(\alpha k + \beta)}$$

…(1)

Here $\alpha, \beta \in C, \operatorname{Re}(\alpha) > 0, \operatorname{Re}(\beta) > 0$,$(a_j)_{km}$, $(b_j)_{kn}$ are the pochammer symbols and m,n are non-negative real numbers.

Equivalence

In this section we shall show the equivalence of single Dirichlet average of $\underset{p,q;m,n}{\overset{\alpha,\beta}{M}}(x)$ function ($k = 2$) with the fractional derivative i.e.

$$S(\beta, \beta'; x, y) = \frac{\Gamma(\beta + \beta')}{\Gamma\beta} (x-y)^{1-\beta-\beta'} D_{x-y}^{-\beta'} \underset{p,q;m,n}{\overset{\alpha,\beta}{M}}(x)(x-y)^{\beta-1} \tag{3.2}$$

Proof:

$$S(\beta, \beta'; x, y) = \sum_{k=0}^{\infty} \frac{(a_1)_{km} \dots (a_p)_{km}}{(b_1)_{kn} \dots (b_q)_{kn}} \frac{z^k}{\Gamma(\alpha k + \beta)} R_n(\beta, \beta'; x, y)$$

$$= \sum_{k=0}^{\infty} \frac{(a_1)_{km} \dots (a_p)_{km}}{(b_1)_{kn} \dots (b_q)_{kn}} \frac{1}{\Gamma(\alpha k + \beta)} \frac{\Gamma(\beta + \beta')}{\Gamma\beta\, \Gamma\beta'} \int_0^1 [ux + (1-u)y]^k u^{\beta-1} (1-u)^{\beta'-1} du$$

Putting $u(x-y) = t$, we have,

$$= \sum_{k=0}^{\infty} \frac{(a_1)_{km} \ldots (a_p)_{km}}{(b_1)_{kn} \ldots (b_q)_{kn}} \frac{1}{\Gamma(\alpha k + \beta)} \frac{\Gamma(\beta+\beta')}{\Gamma\beta\, \Gamma\beta'} \int_0^{x-y} [t+y]^k \left(\frac{t}{x-y}\right)^{\beta-1} \left(1 - \frac{t}{x-y}\right)^{\beta'-1} \frac{dt}{x-y}$$

On changing the order of integration and summation, we have

$$= (x-y)^{1-\beta-\beta'} \frac{\Gamma(\beta+\beta')}{\Gamma\beta\, \Gamma\beta'} \int_0^{x-y} \sum_{k=0}^{\infty} \frac{(a_1)_{km} \ldots (a_p)_{km}}{(b_1)_{kn} \ldots (b_q)_{kn}} \frac{1}{\Gamma(\alpha k + \beta)} [t+y]^k \, (t)^{\beta-1} (x-y-t)^{\beta'-1} dt$$

Or

$$= (x-y)^{1-\beta-\beta'} \frac{\Gamma(\beta+\beta')}{\Gamma\beta\, \Gamma\beta'} \int_0^{x-y} \overset{\alpha,\beta}{\underset{p,q;m,n}{M}} \; (t)^{\beta-1} (x-y-t)^{\beta'-1} dt$$

Hence, by the definition of fractional derivative, we get

$$S(\beta, \beta'; x, y) = (x-y)^{1-\beta-\beta'} \frac{\Gamma(\beta+\beta')}{\Gamma\beta} D_{x-y}^{-\beta'} \overset{\alpha,\beta}{\underset{p,q;m,n}{M}} \; (x)(x-y)^{\beta-1}$$

This completes the Analysis.

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